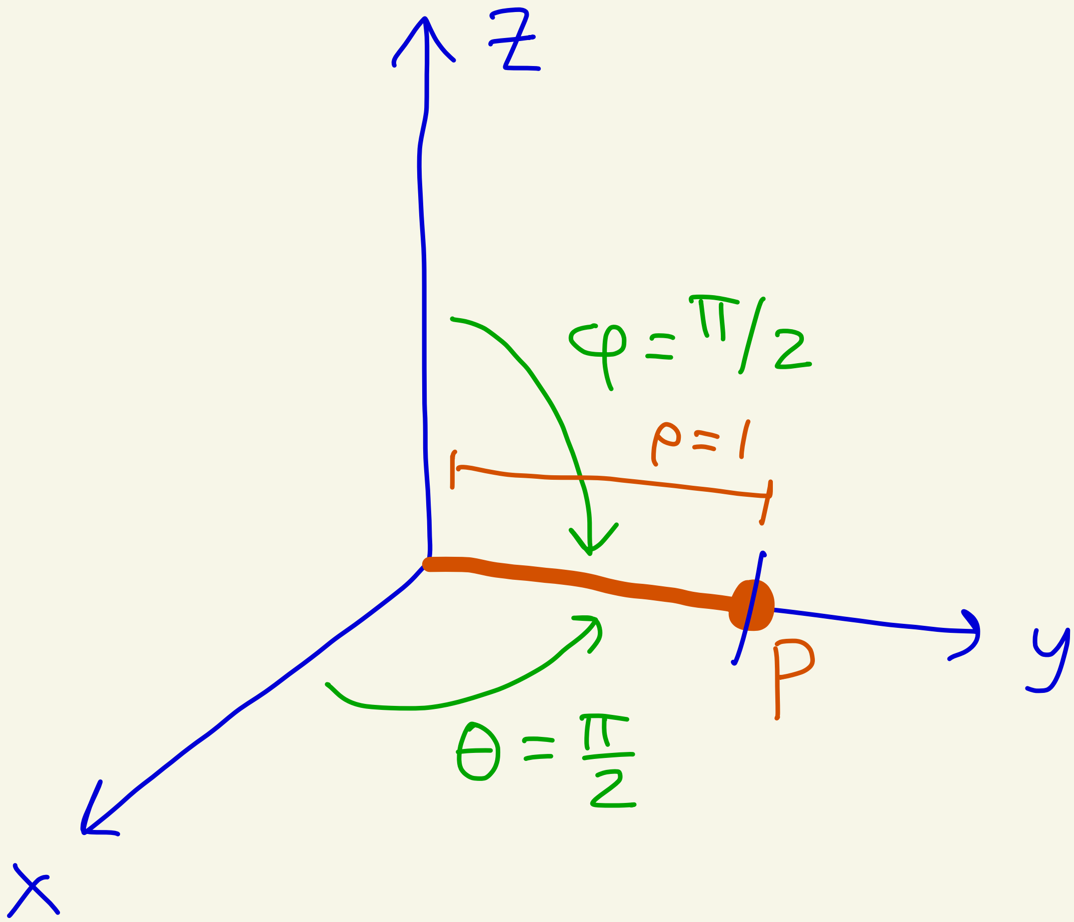
Math 2130
HW 7 Solutions

①

$$(\rho, \varphi, \theta) = \left(1, \frac{\pi}{2}, \frac{\pi}{2}\right)$$



The cartesian coordinates are:

$$(x, y, z) = (0, 1, 0)$$

You can also get them from:

$$x = \rho \sin(\varphi)\cos(\theta) = 1 \cdot \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} \cdot \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} = 0$$

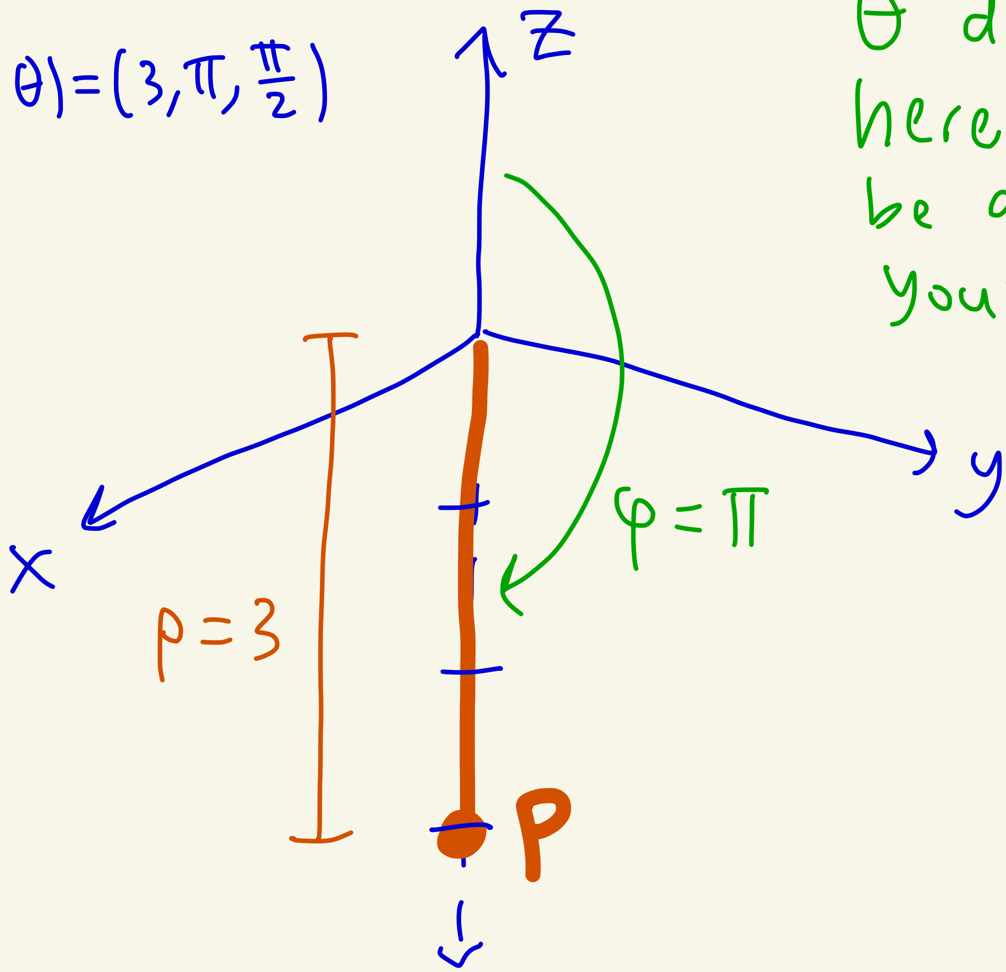
$$y = \rho \sin(\varphi)\sin(\theta) = 1 \cdot \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} \cdot \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} = 1$$

$$z = \rho \cos(\varphi) = 1 \cdot \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} = 0$$

②

$(\rho, \varphi, \theta) = (3, \pi, \frac{\pi}{2})$

θ doesn't matter here. It could be anything and you'd get the same point P



The cartesian coordinates are:

$(x, y, z) = (0, 0, -3)$

You can also get them from:

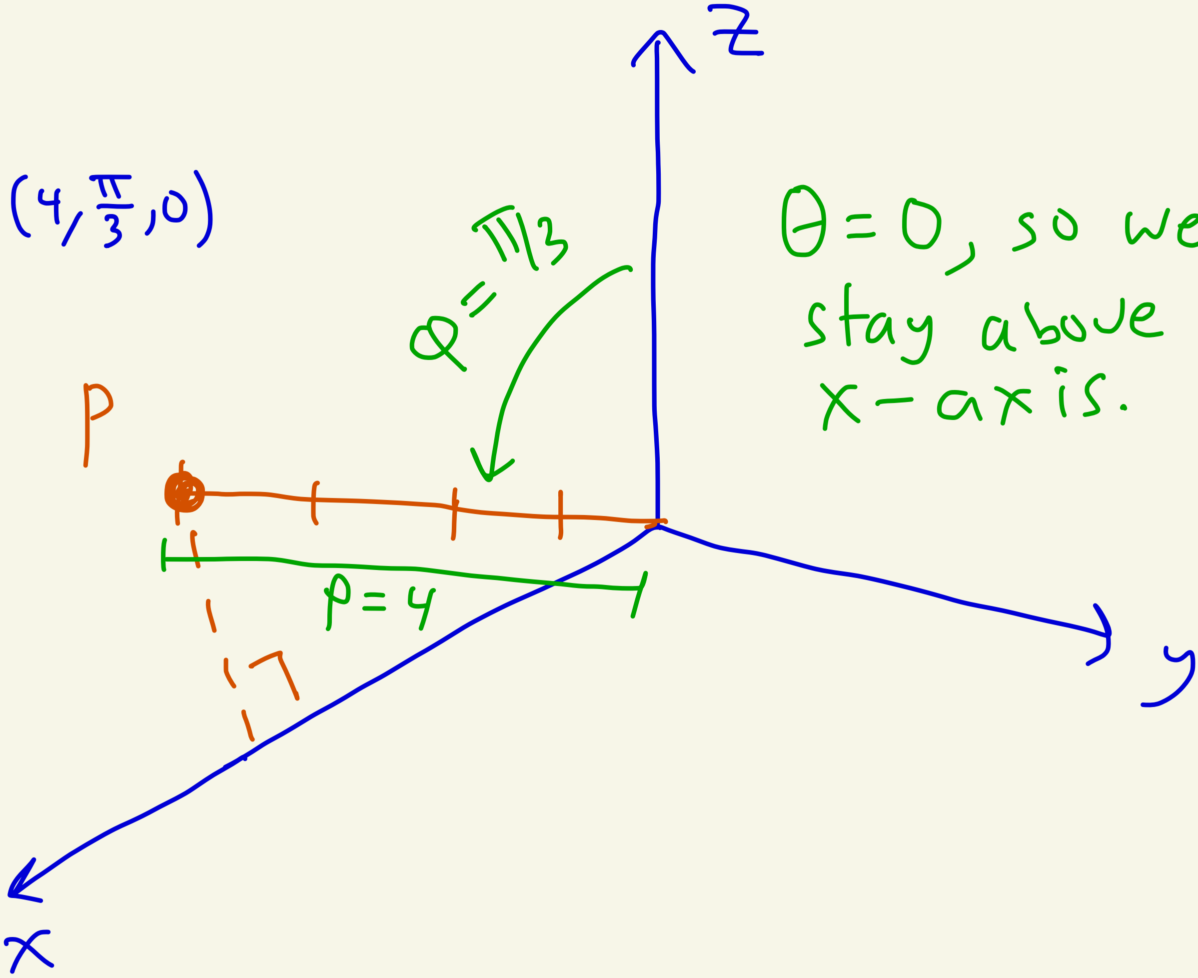
$$x = \rho \sin(\varphi)\cos(\theta) = 3 \cdot \underbrace{\sin(\pi)}_{0} \cdot \cos\left(\frac{\pi}{2}\right) = 0$$

$$y = \rho \sin(\varphi)\sin(\theta) = 3 \cdot \underbrace{\sin(\pi)}_{0} \cdot \sin\left(\frac{\pi}{2}\right) = 0$$

$$z = \rho \cos(\varphi) = 3 \cdot \underbrace{\cos(\pi)}_{-1} = -3$$

③

$(\rho, \varphi, \theta) = (4, \frac{\pi}{3}, 0)$



$\theta = 0$, so we stay above the x-axis.

The Cartesian coordinates are gotten from:

$$x = \rho \sin(\varphi)\cos(\theta) = 4 \underbrace{\sin\left(\tfrac{\pi}{3}\right)}_{\sqrt{3}/2} \cdot \underbrace{\cos(0)}_{1} = 2\sqrt{3}$$

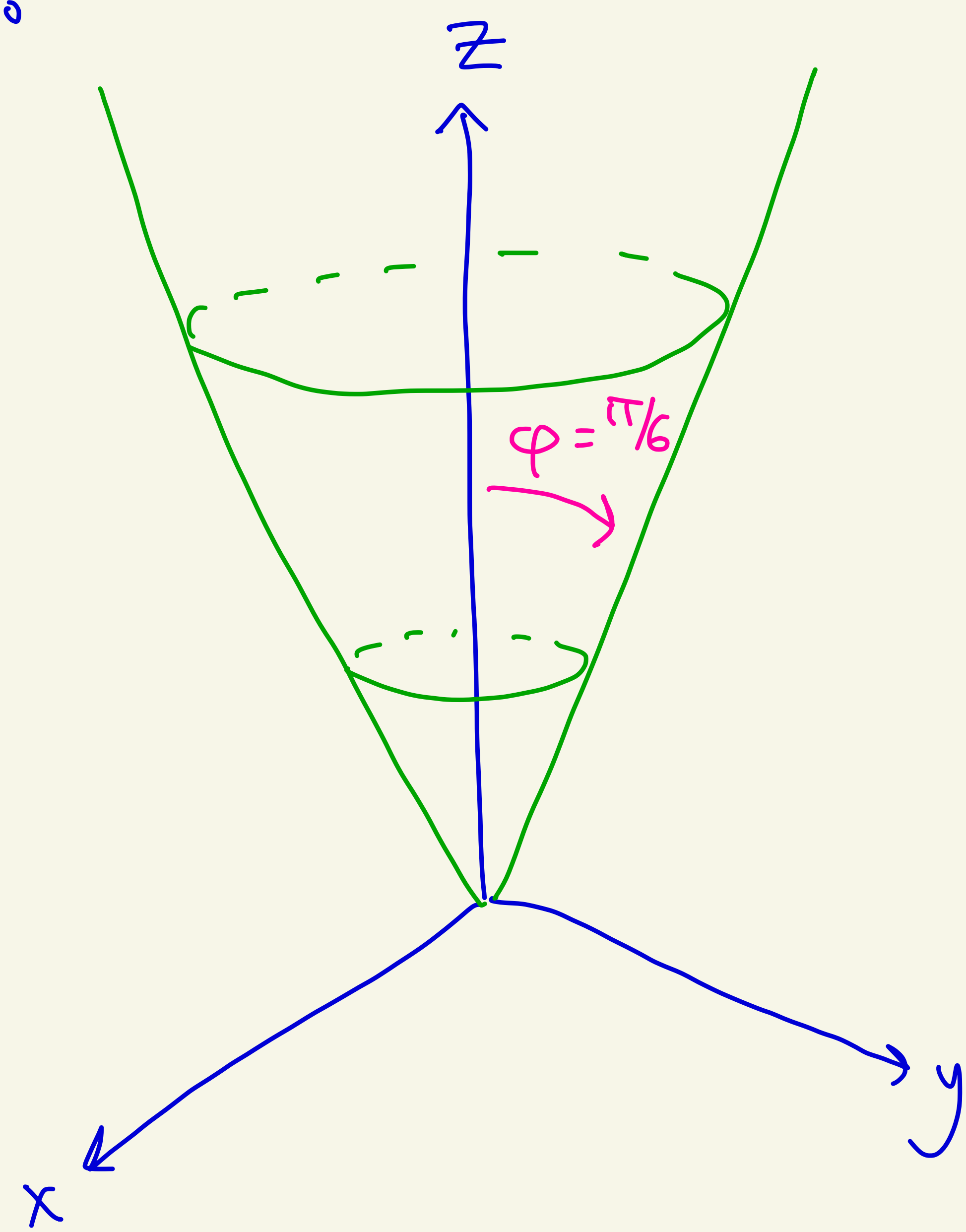
$$y = \rho \sin(\varphi)\sin(\theta) = 4 \underbrace{\sin\left(\tfrac{\pi}{3}\right)}_{\sqrt{3}/2} \cdot \underbrace{\sin(0)}_{0} = 0$$

$$z = \rho \cos(\varphi) = 4 \underbrace{\cos\left(\tfrac{\pi}{3}\right)}_{1/2} = 2$$

$$(x, y, z) = (2\sqrt{3}, 0, 2)$$

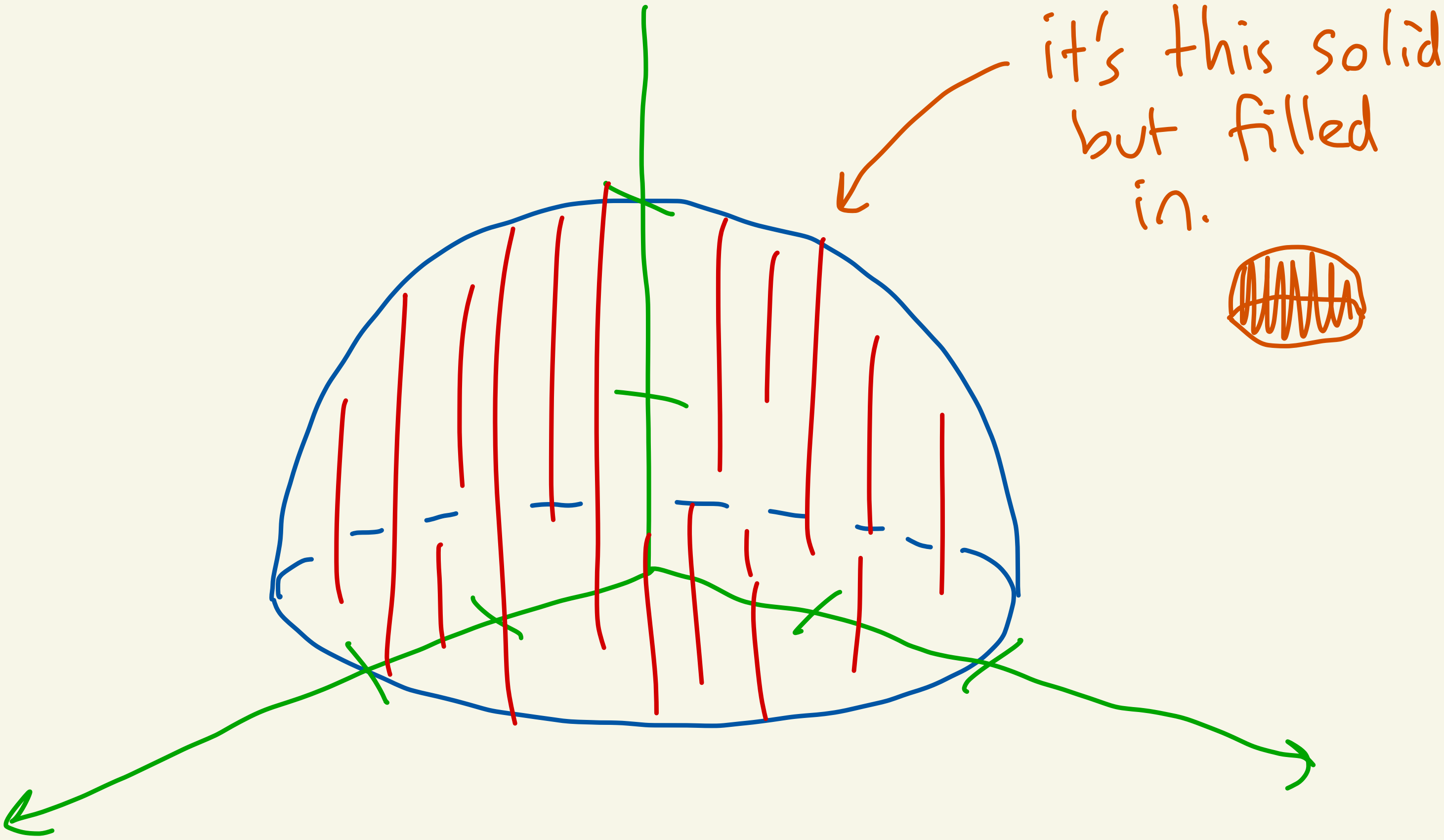
④ $\varphi = \pi/6$

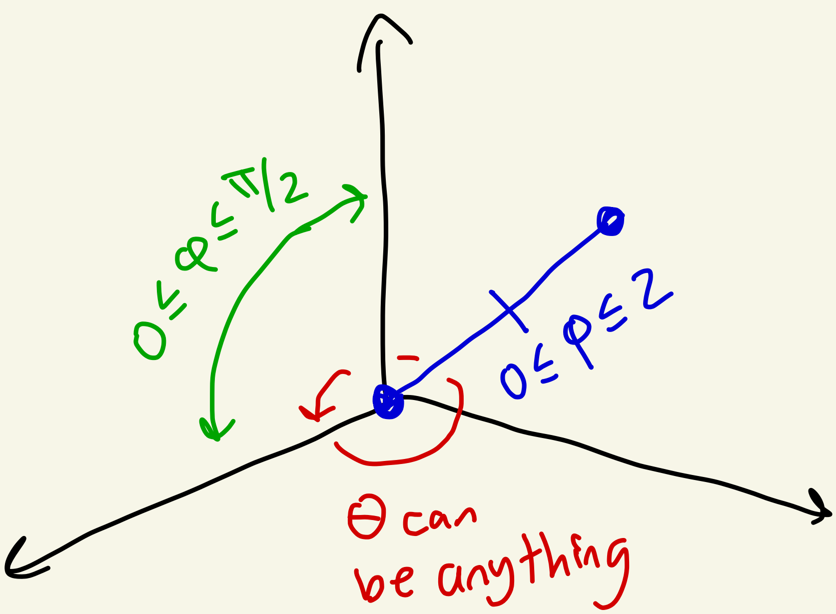
$\varphi = 30°$



⑤ $\rho \le 2$

$0 \le \varphi \le \pi/2$





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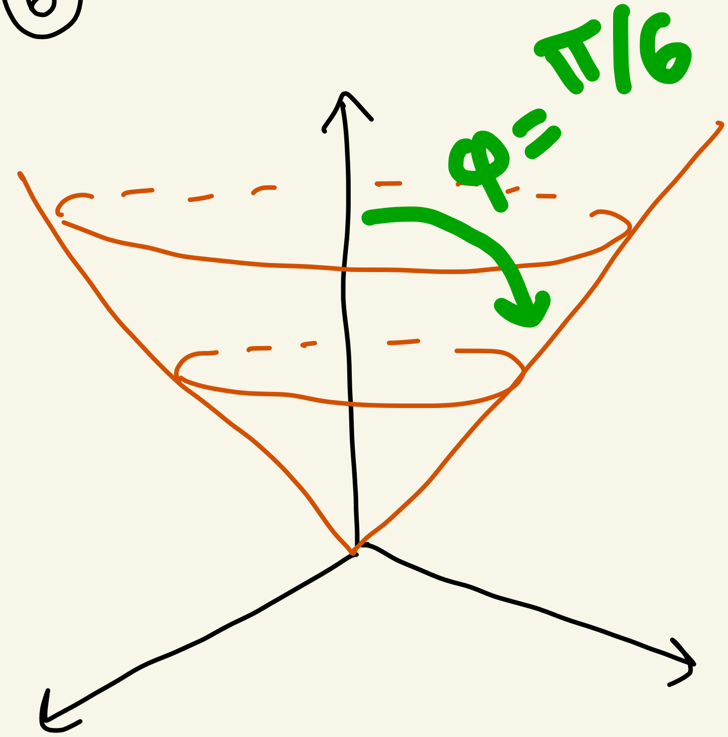
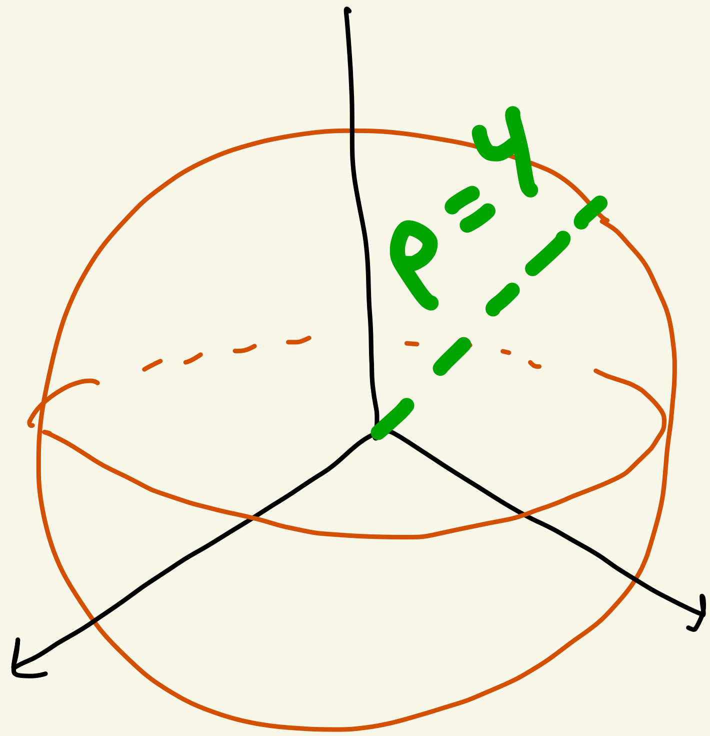
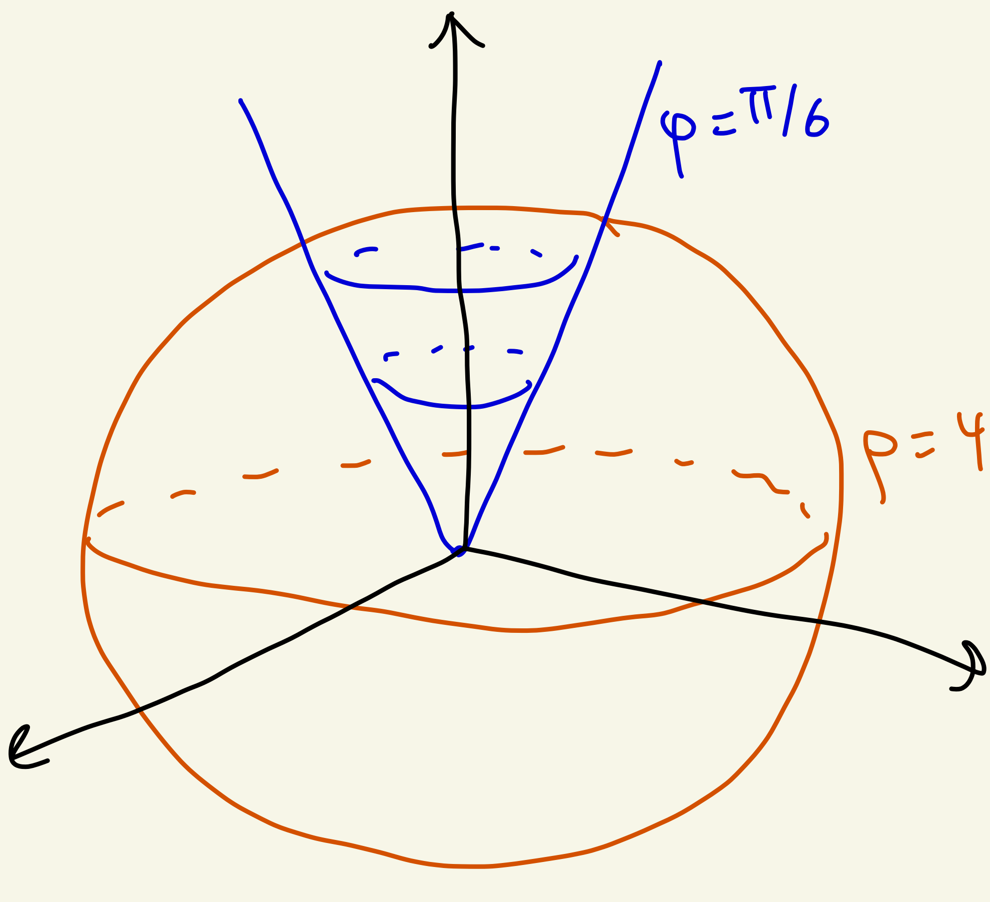
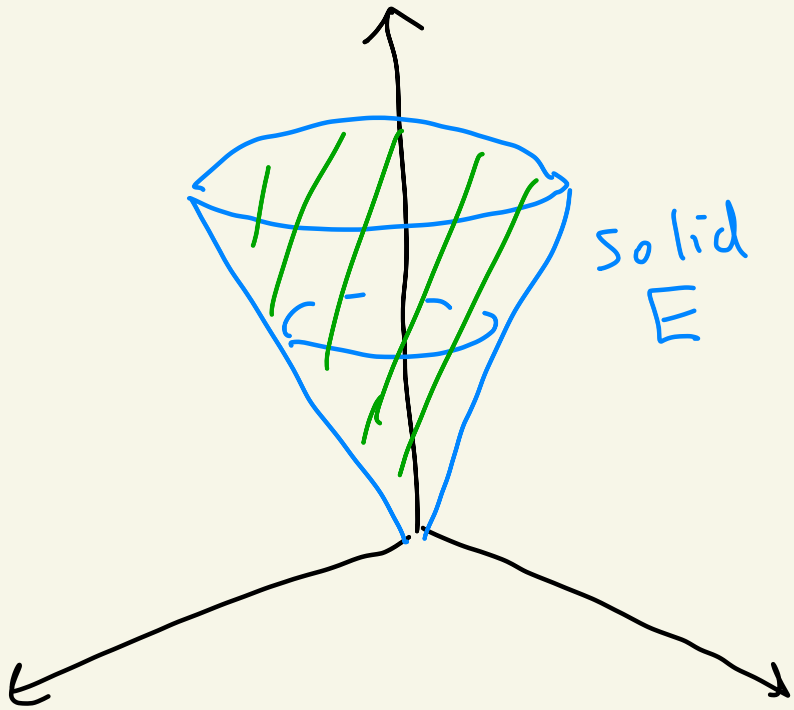








Solid E is parameterized by

$0 \le \rho \le 4$

$0 \le \varphi \le \pi/6$

$0 \le \theta \le 2\pi$

$$\text{Volume of } E = \iiint_E 1 \cdot dV$$

$$= \int_0^{2\pi} \int_0^{\pi/6} \int_0^4 1 \cdot \underbrace{\rho^2 \sin(\varphi)\, d\rho\, d\varphi\, d\theta}_{dV}$$

$$= \int_0^{2\pi} \int_0^{\pi/6} \left[\frac{1}{3}\rho^3 \Big|_{\rho=0}^{4} \right] \sin(\varphi)\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi/6} \frac{4^3}{3} \sin(\varphi)\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \left[-\frac{64}{3}\cos(\varphi) \Big|_{\varphi=0}^{\pi/6} \right] d\theta$$

$$= \int_0^{2\pi} \left[-\frac{64}{3} \left(\underbrace{\cos(\pi/6)}_{\sqrt{3}/2} - \underbrace{\cos(0)}_{1} \right) \right] d\theta$$

$$= \int_0^{2\pi} -\frac{64}{3} \left(\frac{\sqrt{3}}{2} - 1 \right) d\theta$$

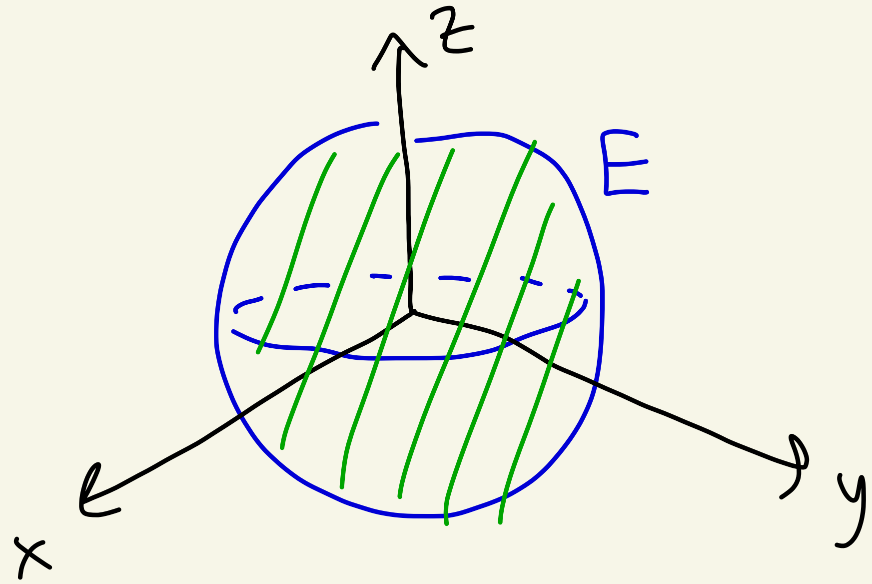
$$= -\frac{64}{3} \left(\frac{\sqrt{3}}{2} - 1 \right) \theta \Big|_0^{2\pi}$$

$$= -\frac{64}{3} \left(\frac{\sqrt{3}}{2} - 1 \right) (2\pi - 0)$$

$$= -\frac{64}{3} \left(\frac{\sqrt{3}}{2} - 1 \right) (2\pi)$$

$$= \boxed{-\frac{128}{3} \pi \left(\frac{\sqrt{3}}{2} - 1 \right)} \approx \boxed{17,958\ldots}$$

(7)



$$x^2+y^2+z^2 \le 1$$

$$\rho^2 \le 1$$

$$0 \le \rho \le 1$$

$$0 \le \varphi \le \pi$$

$$0 \le \theta \le 2\pi$$

density of E

$$= \iiint_E f(x,y,z)\, dV$$

$$= \iiint_E (x^2+y^2+z^2)\, dV$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^2 \cdot \overbrace{\rho^2 \sin(\varphi)\, d\rho\, d\varphi\, d\theta}^{dV}$$

$$\boxed{\rho^2 = x^2+y^2+z^2}$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^4 \sin(\varphi)\, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{\rho^5}{5} \Big|_{\rho=0}^{1} \sin(\varphi)\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi} \left(\frac{1}{5} - 0\right) \sin(\varphi)\, d\varphi\, d\theta$$

$$= \frac{1}{5} \int_0^{2\pi} \left(-\cos(\varphi) \Big|_{\varphi=0}^{\pi} \right) d\theta$$

$$= \frac{1}{5} \int_0^{2\pi} \underbrace{\left(-\cos(\pi) - (-\cos(0)) \right)}_{-(-1)+1=2} d\theta$$

$$= \frac{1}{5} \int_0^{2\pi} 2\, d\theta = \frac{1}{5} \left[2\theta \Big|_0^{2\pi} \right]$$

$$= \frac{1}{5} \left[2(2\pi) - 2(0) \right] = \boxed{\frac{4\pi}{5}}$$

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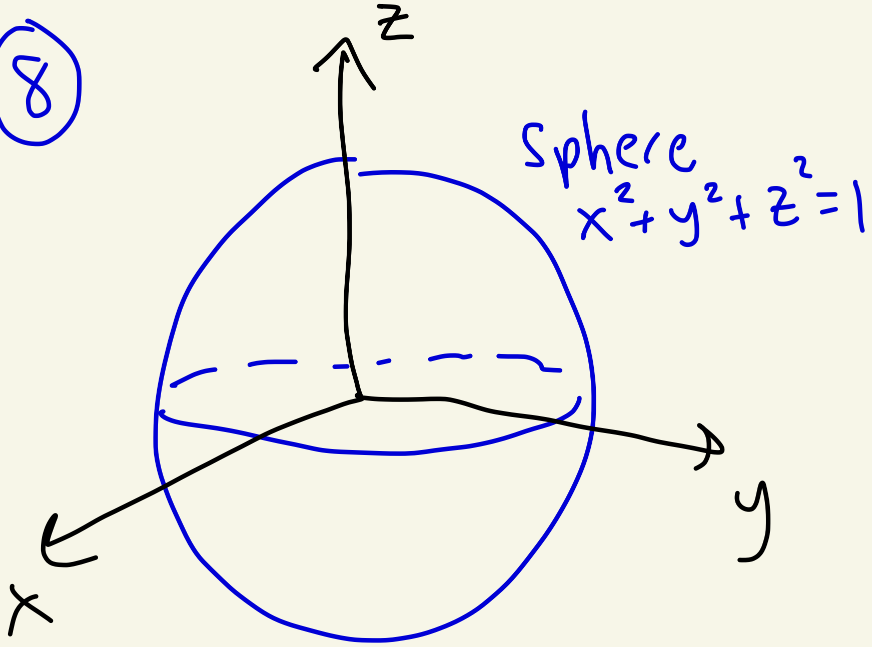

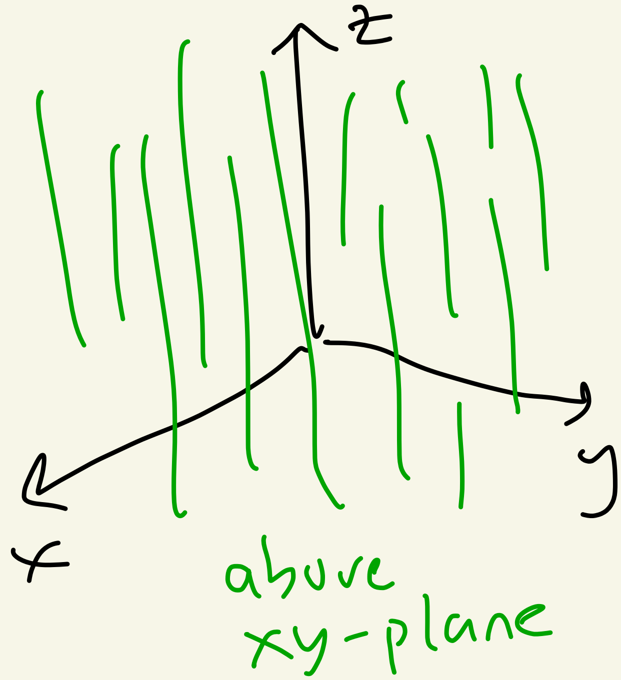

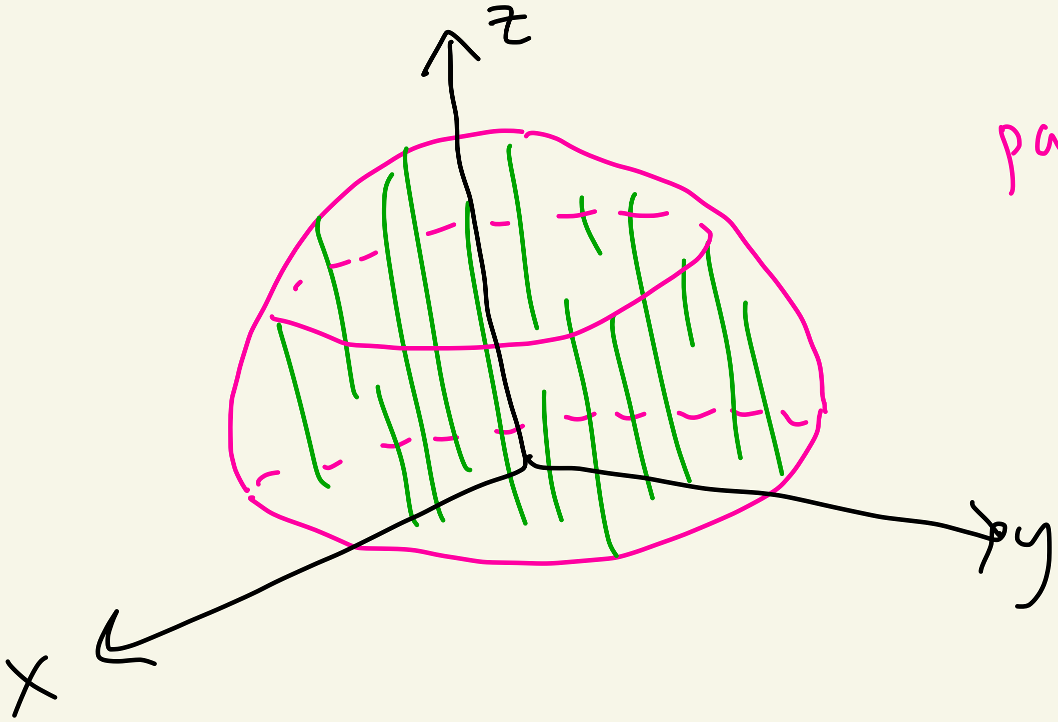

parameterized by

$0 \leq \rho \leq 1$

$0 \leq \varphi \leq \pi/2$

$0 \leq \theta \leq 2\pi$

density of E is

$$\iiint_E \sqrt{x^2+y^2+z^2}\, dV$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \int_0^1 \rho \cdot \underbrace{\rho^2 \sin(\varphi)\, d\rho\, d\varphi\, d\theta}_{dV}$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \int_0^1 \rho^3 \sin(\varphi)\, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \frac{1}{4} \rho^4 \Big|_{\rho=0}^{1} \sin(\varphi)\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \frac{1}{4} (1-0) \sin(\varphi)\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \frac{1}{4} \sin(\varphi)\, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \left[-\frac{1}{4} \cos(\varphi) \Big|_{\varphi=0}^{\pi/2} \right] d\theta$$

$$= \int_0^{2\pi} \left(-\frac{1}{4} \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} - \left(-\frac{1}{4} \underbrace{\cos(0)}_{1} \right) \right) d\theta$$

$$= \int_0^{2\pi} \frac{1}{4}\, d\theta = \frac{1}{4}\theta \Big|_0^{2\pi} = \frac{1}{4}(2\pi - 0) = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$$